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Sagnac Effect and SET Error based Pseudorange Modeling for GPS Applications

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Abstract

Radio Frequency (RF) signals broadcast by Global Positioning System (GPS) satellites have become an indispensable part of military and civil applications, right from missile guidance and civil aviation to farming and surveying. But the critical services provided by the GPS demand millimeter level positional accuracy. Over the years, research has been conducted and it is realized that the positional inaccuracy is mainly due to error in the pseudorange. The pseudorange is derived from the signal emission time (SET) of the satellite (known location) and signal reception time at the receiver (unknown location). Hence the better receiver position accuracy can be achieved by precise estimation of SET and accurate satellite position. If these errors are not corrected, they lead to inaccurate satellite position, which will propagate directly in to the range measurements. In this paper, an algorithm is proposed to estimate the SET and correct the satellite position for sagnac effect. Here the error in SET is modeled by taking into account the satellite clock aging parameters and the elliptical trajectory of the satellite, similarly the sagnac effect is corrected considering the earth's rotation during the signal propagation time. The implementation of the proposed algorithm for the ephemeris data collected from the Dual Frequency (DF) GPS receiver located in Indian subcontinent proved better positional accuracy. The SET and sagnac impact are assessed for all the tracked satellites, but the impact analysis pertaining to only SV PRN07 (Space Vehicle Pseudorandom Noise) are presented in this paper. The proposed algorithm can be implemented to achieve higher accuracy for navigation applications like civil aviation, category I/II Precision Approach (PA) aircraft's landing and Real Time Kinematic (RTK) positioning.

Keywords: Signal Emission Time; Eccentric correction; Satellite clock offset; Sagnac effect; Satellite Solution; Receiver position.

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1. Introduction

The Global positioning system (GPS) is one such satellite navigation system which extensively uses the RF signal broadcast by satellites revolving around the earth. As the navigation and tracking applications become critical they demand higher positional accuracy. Many error sources affect the navigation solution, like the errors originating at the satellite (SET error and Sagnac effect), at the propagation medium (tropospheric delay and ionospheric delay) and at the receiver (multipath error and instrument biases). Among these, the errors originating at satellite are common to all GPS receiver on or above the earth surface. In this paper, an algorithm is proposed to estimate and correct the SET error and error due to sagnac effect. But SET is impaired by two parameters, satellite clock error and eccentric correction. The satellite clock error is due to non ideal atomic clocks onboard the satellite and the eccentric correction is due to elliptical trajectory of the satellite. The former is caused by aging of the clock and the latter is due to space perturbation leading to non circular orbits. Whereas the sagnac effect is due to earth's rotation (rotation of receiver located on earth) during the signal propagation time. This implies the rotation of the Earth Center Earth Fixed (ECEF) coordinate system which is reference coordinate system for receiver position measurement.

The proposed algorithm is implemented on the data collected from a GPS receiver in the Indian subcontinent. The ephemerides are collected on 7th April 2015 from dual frequency GPS receiver (DL-V3 model, make Novatel) located at Department of Electronics and Communication Engineering, Andhra University College of Engineering, Visakhapatnam (17.73^{0}N/83.319^{0}E). During the observation period, the ephemerides data is obtained at 15 seconds epoch interval. During the observation minimum of 9 satellites are tracked in each epoch. Though the analysis is done for all the satellites, only the results pertaining to Pseudo Random Noise (PRN) code 07 are presented in this paper. The impact analysis done in this paper will also be a valuable aid for GPS augmented systems like GPS Aided GEO Augmented Navigation (GAGAN) and Wide Area Augmentation System (WAAS).

2. SET error and Sagnac effect Assessment

GPS positioning requires the user to track the satellites in view in order to receive the broadcast RF signal and derive Coarse/Acquisition (C/A) code ranges (pseudoranges)[1] between user antenna and each of the visible satellites. For precise navigation and tracking, the pseudorange needs to be corrected for SET error, propagation path delays, receiver instrument biases etc[2]. Among these errors, the SET error has a major impact, not only on range as shown in Eq.(1)[3], but also on the satellite position.

$$P_i = \rho + c \times \Delta t_{SET} + \varepsilon \tag{1}$$

Here, P_i is the pseudorange from receiver to i^{th} satellite (m), ρ is geometric range (m), Δt_{SET} is SET error (s), c is the velocity of light (m/s) and ε is the error due to propagation path delays and instrument bias (m). From Eq.(1), it is evident that SET error of 1 microsecond will lead to 300 meters error in pseudorange[4]. The SET error is a result of highly stable but non ideal onboard atomic clock's lack of synchronization with GPS time[5] and eccentric correction is due to the elliptical shape of the satellite's orbits resulting in non zero eccentricity[6].

2.1. Signal Emission Time (SET) error correction

The L_1 RF signal[7] transmitted by each satellite travels pseudorange of distance 'P' meters at light speed 'c' (meter/sec). This propagation time and the signal reception time at receiver as given by Eq.(2)[8] results in the residual time which is the instant at which the signal started from the satellite.

$$t_s = t_{rx} - (P/c) \tag{2}$$

Where, t_s is satellite time of signal transmission (s) and t_{rx} is signal reception time at receiver (s). But the above calculated satellite time of signal transmission is not the accurate SET and will make the GPS navigation and tracking ineffective. Correct assessment of SET and its error is important inorder to get higher positional accuracy anywhere on or above the earth[9]. To estimate the SET error, the satellite time of signal transmission (t_s)[10] is calculated using Eq.(2) and time of clock (t_{oc}) is read from satellite ephemerides. The clock offset (dt)[11] calculated using Eq.(3)[12], and atomic clock bias and aging corrections parameters[13] bias (a_0) (sec), drift (a_1) (sec/sec) and drift rate (a_2) (sec/sec^2) put in second order polynomial form Eq.(4) gives the satellite clock error (ε^{sc}),

$$dt = t_s - t_{oc} \tag{3}$$

$$\varepsilon^{sc} = a_0 + a_1 dt + a_2 dt^2 \tag{4}$$

Satellite clock error free time of signal transmission is the correct time when the signal was emitted by the satellite, but this time parameter is made even more precise by eccentric correction given by Eq.(5)[14].

The satellite orbits are not circular, which implies non zero eccentricity. Due to their elliptical shape, eccentricity adds an error to Signal Emission Time (SET). Even an eccentricity as small as 0.01 makes the satellite to change altitude periodically and leads to 23 nanoseconds error in SET[6]. The Eccentric correction (Δt_{ecc}) is computed using Eq.(5)[6],

$$\Delta t_{ecc} = (-2\sqrt{\mu}/c^2)e\sqrt{a}\ \sin(E_k) \tag{5}$$

Where, Δt_{ecc} is eccentric correction error (s), μ is earth's gravitation constant (3.986005 $\times$ 10^{14}m^3/s^2), c is speed of light = 3 $\times$ 10^8(m/s), *e is e*ccentricity, *a is s*atellite orbit semi major axis (m) and E_k is eccentric anomaly of satellite orbit (rad). The signal emission time (SET_{GPS}) which is in synchronization with GPS time is computed using Eq.(6). Using orbital estimation algorithm and broadcast orbital parameters, the satellite positions are estimated for this instant of time.

$$SET_{GPS} = t_s - \Delta t_{SET} \qquad \text{Where, SET error } (\Delta t_{SET}) = \varepsilon^{sc} + \Delta t_{ecc} \tag{6}$$

In the above equation, t_s is satellite time of signal transmission (sec), ε^{sc} is satellite clock error (sec) and Δt_{ecc} is eccentric correction error (sec).

2.2. Sagnac Effect Correction

Each GPS satellite broadcasts 50 bits per second navigation data message containing orbital parameters. Such messages of at least four satellites are utilized by the receiver to compute the receiver position and velocity at given instant of time. But the first step in estimating the receiver position is to compute the satellite position coordinates. In GPS both the satellite position and the receiver position coordinates are expressed in Earth centered Earth Fixed (ECEF) cartesian coordinate system, which is defined by World Geodetic System 1984 (WGS84).

From each satellite to the receiver, the pseudorange is calculated, which is the difference of signal reception time at the receiver and the signal transmission time at the satellite, multiplied by speed of light. In the GPS system, time is the fundamental parameter for the all the calculations. These calculations are done assuming a non rotating Earth – centred ECEF coordinate frame. But as the earth is rotating around its axis, the receiver located on the earth experiences the uniform motion as that of the earth's rotation. Motion of the earth and hence the receiver during the signal propagation from satellite to receive implies change in location of the receiver during signal propagation time (Δt_{ROT}), this phenomenon is known as sagnac effect [15] and this needs to be accounted in order to achieve the precise navigation solution. The sagnac effect is corrected by transforming the ith satellite coordinates $x_i^s = [\,x_i^s, y_i^s, z_i^s\,]$ which are calculated in ECEF coordinate frame at the signal emission time (SET_{GPS}) to the satellite coordinates $x_i^R = [\,x_i^R, y_i^R, z_i^R\,]$ in ECEF coordinate frame[16] at signal reception time (t_{rx}).

$$\Delta t_{ROT} = t_{rx} - SET_{GPS} \tag{7}$$

$$X_i^R = M_{ROT}(\omega_E \times \Delta t_{ROT}) \bullet X_i^s \tag{8}$$

Where, ' ω_E ' is earth's rotation rate which is 7.2921151467 $\times$ 10^{-5}rad/sec and the rotation matrix is given by,

$$M_{ROT}(\omega_E \times \Delta t_{ROT}) = \begin{bmatrix} \cos(\omega_E \times \Delta t_{ROT}) & \sin(\omega_E \times \Delta t_{ROT}) & 0 \\ -\sin(\omega_E \times \Delta t_{ROT}) & \cos(\omega_E \times \Delta t_{ROT}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The rotated satellite position and the sagnac affected satellite coordinates coincide with z-axis .Hence the zero error is observed in z-axis. The sagnac effect corrected satellite positions x_i^R are used for the receiver position estimation.

3. Results and discussion

Statistical analysis of the result shows that SET error and Sagnac effect are too large to neglect. This error is estimated for typical geographical location in Indian subcontinent over the Bay of Bengal for typical ephemerides collected on 7th April 2015 from the DFGPS receiver located in Department of ECE, AUCE, Visakhapatnam (Lat:17.73^{0}N/Long:83.319^{0}E), India. The data collected are in RINEX 2.10 format.

The impact of SET error and the sagnac effect on orbital solution and Navigation solution are analysed and relevant graphs and tables are presented in this paper. During observation period, out of 32 satellites, a minimum of 9 satellites were visible in each epoch. Though the errors are computed and analyzed for all the visible satellites, the errors pertaining only to SV PRN07 are presented in this paper, which was tracked for about 8 hours. The navigation solution for each epoch is calculated using all the visible satellites.

Table I: Impact of SET error and Sagnac effect on pseudorange of SV PRN07 as tracked by DFGPS receiver at AUCE on 07th April 2015

	Satellite SV PRN07		
	Signal Emission Time (SET) Error [micro seconds]	***Pseudorange Error due to SET [meters]***	***Pseudorange Error due to Sagnac Effect [meters]***
Min	5.3604	1607	1557
Max	5.4004	1619	1627
Mean	5.3804	1613	1592
Standard Deviation	0.0090029	2.699	18.36

Table I details the SET error and the sagnac effect on SV PRN07, the results show that the expected time delay in signal emission is 5.3804 microseconds and that intern affected pseudorange by 1613 meters. The sagnac effect also introduced the pseudorange error of 1592 meters. The change in pseudorange error over the observation period of 8 hours is shown in Fig.(1) and Fig.(2). The figures show that the impact of SET error on pseudorange is uniform and continuous, where as the pseudorange error due to sagnac effect is high during first few hours of tracking.

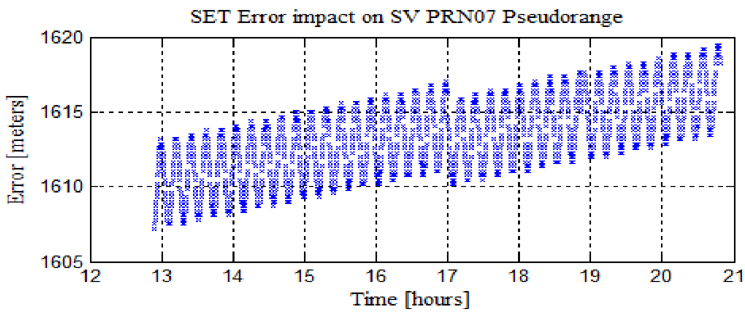

Fig.1 SV PRN07 Pseudorange Error due to Signal Emission Time (SET)error

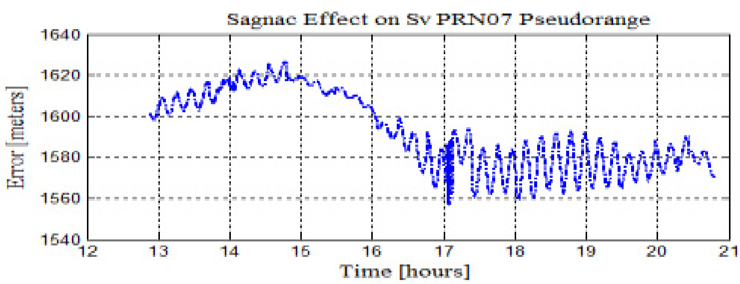

Fig.2 Sagnac effect on Pseudorange of SV PRN07

The values of table II show that due to sagnac effect SV PRN07 experienced a minimum error of 34.99 meters in x-axis and –160.5 meters error in y-axis. The error in z coordinate is zero because z-axis of the earth fixed ECEF coordinate frame and the coordinate frame rotated to counter the sagnac effect, coincide with each other. Due to sagnac effect the estimated satellite position is about 118 meters farther away from the true position.

Table II : SV PRN07 Orbital Solution Error and Navigation Solution Accuracy Measures of DFGPS receiver Located at AUCE on 07th April 2015

	SV PRN07 Orbital solution error [meters]			*DFGPS Navigation solution error [meters]*			*Navigation solution Accuracy Measures [meters]*	
	x-	*y-*	*z-*	*x-*	*y-*	*z-*	**DRMS(2D)**	9.0151
Min	34.99	-160.5	0	6.788	82.8	21.61	**2DRMS(2D)**	19.0301
Max	132.4	51.38	0	25.89	128.1	43.14	**CEP(2D)**	7.2057
Mean	87.44	-30.7	0	15.92	108.6	33.87	**MRSE(3D)**	9.6382
Standard Deviation	36.2	68.54	0	3.834	8.159	3.409	**2DRMS(3D)**	19.2764
Maximum Distance	118.6697			28.21			**SEP(3D)**	7.8552

The positional error in x-,y- and z- coordinates of the satellite are shown in Fig.(3). The errors due to sagnac effect, for the entire visible period are shown in the graph. Similar positional errors are observed in all the visible satellites. The navigation solution is calculated considering all the tracked satellites. This means the positional errors of all the satellites contributed to degraded position of the receiver. The receiver position errors are shown graphically in Fig.(4). For the mentioned DFGPS receiver the position errors are 6.788 meters, 82.2 meters and 21.61 meters respectively in x-, y- and z- coordinates, for the minimum errors in satellite position and pseudorange. The navigation solution accuracy measures in 2D (Horizontal) and 3D (Horizontal and Vertical) show that 50% of the 2D errors are within (CEP) 7.2057 meters and 3D errors are within (SEP) 7.8552 meters of its expected value.

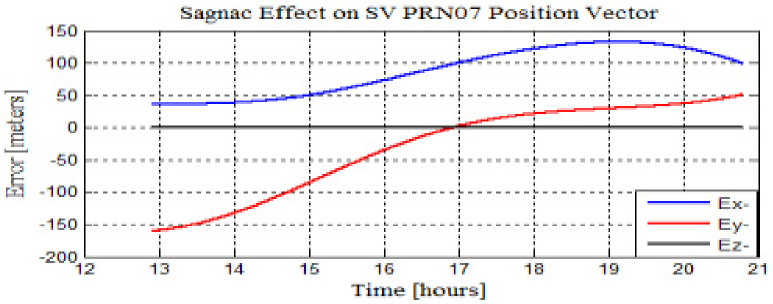

Fig.3 SV PRN07 Satellite position inaccuracy due to Sagnac Effect

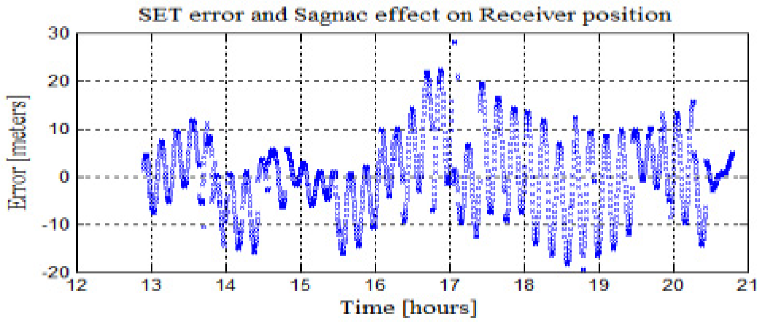

Fig.4 Error in receiver position due to Signal Emission Time error and Sagnac effect

4. Conclusions

Accuracy requirements of precise navigation applications like CAT I/II aircraft's landing, civil aviation and missile guidance can be met by correcting the pseudorange. Implementation of the proposed algorithm proves that inaccuracy in signal emission time and the effect of sagnac cannot be ignored. It is found that due to SET error the calculated signal time of transmission from the satellite SV PRN07 is later than the actual time and is in the order of microseconds. The error in satellite position due to sagnac effect also manifests as the pseudorange error. Both the SET error and the sagnac effect lead to SV PRN07 position error farther away 118.6697 meters from its true position. It is observed that the z- coordinate errors are zero because the z- axis of ECEF coordinate frame coincide with that of the rotated coordinate frame and hence due to the sagnac phenomenon error is observed in only x- and y- axis. Such positional inaccuracy of all the tracked satellites collectively influence the navigation solution and it is found to be 28.21 meters farther away from its actual position. The analysis of 2D and 3D errors of navigation solution presented here will be a valuable aid for precise surveying applications, Precise Point Positioning (PPP) and geographic information systems (GIS).

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